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Volume Calculation of Caverns

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abstract

In 2005 the Institute of Geotechnical Engineering and Mine Surveying of Clausthal University of Technology, Germany, has done a study on different methods for volume modeling and volume calculation of sonar measurement data of leached caverns.

About 150 sonar data sets belonging to 75 caverns were applied and computed by 6 different volume and 8 different surface based models. Because the real volume of a cavern is unknown the methods must be compared relatively.

Based on study results and further considerations the paper gives an overview of possible procedures for volume calculation of cavern related sonar data. It will be differentiated between surface based and three-dimensional modeling. In addition an outlook onto alternative statistical and stochastic methods is given.

1 Description of Project

Target of the project was the investigation and comparison of several methods for volume calculation of leached caverns based on sonar data sets. As far as possible an evaluation was done, considering immanent errors, plausibility, applicability as well as the potential of further development. In the end, the results were compared with the results of the method used by SOCON.

2 Approaches to Volume Modeling

The volume respectively the surface area of geometrical primitives can be calculated easily, both for surfaces (e.g. triangle, trapezoid) as well as bodies (e.g. cuboid, frustum). If complex surfaces or bodies can be fragmented completely into geometrical primitives the total volume of the complex surface or body can be calculated by the sum of the primitive volumes. Because of the irregular shape of a leached cavern this approach is practically not applicable or leads to significant errors.

Within the project the following approaches were taken into account:

- surface based modeling
- three-dimensional modeling
- statistical and stochastic modeling.

Modeling like mentioned above means the entirety of definitions and rules for e.g. calculations, estimations, interpolation, approximation or symmetry.

Most approaches of volume modeling are a combination of surface based and three-dimensional modeling, e.g. the method used by SOCON.

The basis for a statistical approach is the description of spatial parameter as a random function by use of autocorrelation of adjacent measuring points. A stochastic approach is based on simulation models, e.g. a probability whether a stochastic point is within a body or not using conditioned relation between stochastic points and measuring points.

Because of the similarity to the method used by SOCON the focus was set on surface based and three-dimensional modeling. Statistical and stochastic modeling would be interesting to investigate in a scientific way (efficiency, plausibility), but seems not significant for engineering purposes.

3 Surface based Modeling

SOCON sonar data format is structured into sections:

- horizontal sections
- vertical sections
- sections defined by plain polygons describing hidden leached pockets.

These sections are generated out of three-dimensional raw data by intensive use of expert knowledge. To establish identical pre-conditions, all methods of calculation tested within the project were using this knowledge based interpretations.

There is no difference between the volume calculation of the sections listed above in principle.

3.1 Method used by SOCON

The method for volume calculation of sections used by SOCON is based on circle segments:

$$A = \sum \frac{\pi r_i^2}{n}$$

n = number of measured radii within a section.

3.2 Linear Interpolation (Gauß)

Linear interpolation is the simplest way to describe the boundary of a section, established into land register by Gauß. Two adjacent measuring points are connected by a straight line and the surface area can be calculated as follows:

$$2A = \sum r_i \, r_{i+1} \sin(\alpha_{i+1} - \alpha_i) \quad .$$

A linear interpolated surface area of a convex boundary segment will be calculated too small systematically. Otherwise, a concave boundary segment will be calculated too large systematically. The significance of these systematic errors depends on the density of measuring points.

3.3 Interpolation and Approximation by Circle Segments

Each three adjacent measuring points can be connected by a circle segment. Then the total boundary of a surface is approximated by several circle segments. The surface area of each circle segment can be calculated by the surface area of the circle segment k_{Sk} and the linear interpolated triangular circle sector as follows:

$$A = \sum 0.5 \, r_i \, r_{i+2} \sin(\alpha_{i+2} - \alpha_i) + k_{S_k}$$

$$k_{S_k} = \frac{h}{6s}(h^2 + 4s^2) \quad .$$

This approach leads to systematical errors as well, because a circle along three points is always of highest curvature. In contrast to the linear interpolation described above convex segments will be calculated to large systematically and concave segments will be calculated to small systematically. The significance of these systematic errors depends on the density and irregularity of measuring points.

3.4 Spline-Interpolation

Adjacent measuring points can be connected by sectional polynomials (NURBS – Non Uniform Rational Basis-Splines). The polynomials are defined by distance and direction to supporting points. This is a quasi-interpolation if the co-ordinates of the supporting points are given by the measuring points and the polynomials approximate the supporting points infinitesimal close. The representation of a boundary by splines can be controlled by conditions of continuously and approximation. Within the project spline were generated by the programs:

- AutoCAD™ and
- Rhinoceros™ .

Both programs control splines by minimizing the gradient along the supporting points. Therefore a convex boundary segment will be calculated too small systematically. Otherwise, a concave boundary segment will be calculated too large systematically. The significance of these systematic errors depends on the density and irregularity of measuring points.

4 Three-dimensional Modeling

4.1 Volume Calculation using Sections

Volume calculation using horizontal sections is possible by several methods. Always the calculated area of a section is multiplied by the average distance to the next horizontal section. The differences of these methods are given by the kind of interpolation between the horizontal sections. This is realized by weighted multiplying of two ore more sections and the distances between.

The method used by SOCON is based on a linear interpolation (cylinder) between the horizontal sections. The so called Trapezoid-Method uses as well a linear interpolation between the horizontal sections but shaped like a trapezoid. The Simpson-Rule is based on a parabola interpolation between the horizontal sections.

A further method using vertical sections is based on a rotation of these sections around the sonar sensor axis. Afterwards the average of the integrated volumes of these rotational bodies will be calculated as follows:

$$V = 2\,\pi\,\Omega\,A \qquad .$$

Although the shape of a vertical section is mostly elongated compared to a horizontal section some similar systematic errors can be expected.

Using cylinder for representation the volume will be calculated too large systematically. Using Frustums the volume will be calculated too small systematically. Are the boundaries represented by a parabola or polynomial interpolation the volume will be calculated too small systematically.

4.2 Kepler Algorithm

Three layers of horizontal sections of a sector can be designed to a barrel like primitive. The barrels volume can be calculated by an inside rotating frustum and an outside rotating cylinder. The boundary is represented by weighted averages of the radii as follows:

$$V_{Barrel} = \frac{1}{9}\pi\,(h_{i+2} - h_i)(r_i{}^2 + 5r_{i+1}{}^2 + r_{i+2}{}^2 + r_i r_{i+1} + r_{i+1} r_{i+2}) \qquad .$$

5 Three-dimensional Volume Calculation

Boundary representation models describe bodies using plains, edges and vertexes. The volume calculation is based on definition and numerical integration of sub-bodies. Such volume models are explicit and easy to solve. Instead of linear interpolation between adjacent measuring points a polynomial interpolation by NURBS is possible.

For both the problem is an automatic generation of topology out of the three-dimensional point cloud containing data from vertical and horizontal sections as well as sections from plain polygons describing hidden leached pockets.

A further method of three-dimensional volume calculation is based on spatial partitioning, e.g. using voxel (volume elements like cubes or cuboids). The result is a discrete image of the volume model. The volume of the total body can be calculated by the voxel sum easily. The accuracy along the boundary depends on the voxel size. Using cubic inch size, what means a sufficient boundary representation, around 30 billion voxel would be fit into a 500.000 m³ cavern. Regarding a PC computing power a compression mode should be implemented (e.g. octree based algorithm)

5.1 Statistical and stochastic Methods

For the representation of complex bodies statistical and stochastic methods should be considered. Two examples are outlined below.

Some statistical methods are based on balancing functions representing the three-dimensional point cloud. The basic idea behind is to represent the point cloud at first by a primitive body, e.g. a cylinder. The shortest distance of each measuring point to the primitive will be calculated. The total sum of all calculated distances will be minimized by primitive deformation and adjustment techniques.

The so called Monte-Carlo-Simulation uses a stochastic approach to calculate the volume of a complex body. The measuring point cloud will be represented by a primitive body, e.g. a cylinder. For a random sample out of a theoretical infinite quantity of three-dimensional points will be calculated if each single point is inside or outside the measuring point cloud. The volume of the measuring point cloud will be estimated to:

$$Volume = \frac{\sum positive\ results}{\sum sample} \times Volume\ of\ primitive\ body \quad .$$

Ones can expect a good approximation for the volume of the measuring point cloud by using a huge number of single simulations.

6 Tests

150 cavern measurements were selected for testing, classified into:

- elongated vertical forming
- elongated horizontal forming
- other horizontal or vertical forming
- similarity to geometrical primitives (e.g. cylinder, sphere, trapezoid)
- indifferent forming.

The following methods for the calculation of section areas were tested:

- linear interpolation
- approximation by circle segments
- approximation by parabola
- polynomial interpolation.

The methods listed above were the basis for volume calculation using:

- cylinder approximation
- trapezoid approximation
- Simpson Rule
- Kepler Algorithm
- rotational method.

Individual tests regarding the roughness of cavern boundaries were done, showing significant differences between the applied methods. Because of project time limitations this aspect should be investigated later. As well, statistical assured statements about the correlation between cavern shape and systematical error of a specific method were not realized within project lifetime.

7 Results

Because the real volume of a cavern is practically unknown, the comparison and evaluation of methods for volume calculation must be based on considerations in principle.

Evaluation criteria are:

- verifiable implausibility
- immanent errors
- significant deviations to the average result of all methods
- algorithm stability
- applicability.

The percentage deviations given below are relatively to the results of SOCON method.

Using spline interpolation as well as approximation by circle segments implausibility occurred in case of strongly varying radii.

Using approximation by circle segments an overlay of adjacent segments is possible. This occurred only in case of hidden leached pockets.

A trapezoid approximation as well as Simpson Rule leads to significant less volume.

An approximation by circle segments leads to indifferent results in volume.

The implementation of spline approximation leads to hardly insignificant results.

In comparison the method used by SOCON shows no implausibility or significant deviations. In addition, the algorithm used by SOCON was stable. The investigation did not indicate that any implementation of the investigated methods into the method used by SOCON would give results more plausible.

More interesting seems a statistical or stochastic approach. Because of the extensive development effort these approaches are justified only, if other methods are not available, to complex or not applicable. This is not given in the above mentioned case.

For ascertained results contact the author please.

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