Summary

For the adjustment and analysis of observations in linear models introducing the well known method of least squares the mathematical geometry of observations holds a decisive role for the examination of the obtained results.
The content of this contribution is an extensive analysis of the inner structures of this observation geometry by revealing specific geometrical features.
They can be determined by the choice either of the adjustment model or the observation arrangement.
To mention one result are a priori relations between the residuals, that are predefined (independently of the actual observation vector). These individual properties are characterized by the so-called latent restrictions (hidden condition equations within observation equations).
This introduced expression represents a generalization of the well-known term of restriction equations for parameters.
The so-called normal form of a design matrix is taken as a basis for practical calculations. In addition an algorithm for the determination of a normal form for the recognition of latent restrictions is presented.

As result (rough analysis of geometry) one obtains several groups of observations characterized by latent restrictions.
They control themselves among each other, but cannot be controlled by the remaining observations.
It is wide spread practice in analysis and adjustment to take into account the so called partial redundancies to be an adequate indicator for the reliability (control) of observations. However they are insufficient for this purpose in general.
On the other hand the reliability of observations among themselves can be indicated for the first time by the identification
of latent restrictions.
From that point of view statistical test or other analysis instruction standards should be modified or extended by new instructions, taking use of this pattern of latent restrictions.

Further topics of the thesis deal with balancing factors, that can be used for a deeper understanding of the underlying geometry of observations – plane to see as a deviation of an ideal geometry from the investigated one. Introducing the extended adjustment model of balanced least squares the degree of freedom is equally shared by all observations.
This idea is taken from the simple arithmetic mean, and extended to the application of more than one unknown
parameter to be determined. It results in partial redundancies that obtain equal numeric values. In the special
adjustment case for just one degree of freedom this procedure of balanced observations applies that the variance of the
unit weight is identical for balanced and not balanced adjustment.
The adjusted observations turn into the original observations.
This property does not hold true for more than one degree of freedom.

For the practical numerical analysis of the geometry of observations a suggestion for the necessary computing activities
is presented. It contains the aspects of partial adjustment (groups within just one model), rank defects, multiple observations, total and zero-redundancy, direct adjustment within an adjustment of indirect observations, disintegrating of the normal form and latent restrictions (rough analysis) up to the calculation of the balancing factors as measure for the
deviation from ideal geometry (fine analysis of observations).
Geodetic examples demonstrate the proposed method of analysis of observation geometry and an interpretation of the latent restrictions in engineering purposes is given.
This extended analysis of the internal structures of observation geometry forms the basis for an extended sight on the optimization of geodetic networks (zero order design to third order design) or the optimization of geodetic measuring
operations.

Further applications of this knowledge depend on the analogies between the adjustment and theory of structures, that
are pointed out within the large field of mechanical networks. A degree of statically undetermination, equally distributed
on all members of a frame structure, corresponds to the even distribution of the total degree of freedom of adjustment on
all observations (balancing).
The mechanical interpretation of the balancing factors related to Hooke's law is analogue to the geodetic network optimization, just adding material properties (optimization of cross sections). In analogy to the statistical interpretation
of variances of the unit weight a view of the strain energy takes place with balancing a single statically undetermined
frame structure. With same material effort the strain energy and the normal forces can be reduced by modification of the cross sections of a member according to the balancing factors.
The practical application of the normal form is indicated during the frame structure calculation. In the special case of just one degree of freedom the balancing factors can be set into direct relationship with the forces.
Thus an interpretation of the balancing factors becomes possible as forces during the law of gravity.
For the analysis of frame structures the recognition of latent restrictions is of large importance, because of the determination of sub-structures and their control (reliability).